

**Title:** Listening to the Trajectory of the Lesson

**Strapline:** How a teacher listens to a class collective will alter the trajectory of their learning

**Summary:** Using a listening taxonomy developed by Davis (1997), teachers can examine the quality of their interactions with students and use the taxonomy to align their lessons hypothetical learning trajectory (Simon, 1995) with their students needs, while at the same time challenging the conception of mathematics as a transcendent body of knowledge through transformative collective sense making. An example of working through the problem of division by 1 is given.

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## 1) Main Section

Based on the constructionist view of learning, the sense each of us makes is based on our own complex histories, and is consistent with, but never identical too, another persons own embodied understanding of that concept. Each stage of listening has different implications for the question, what is mathematics?

It is through listening that we are able to provide the structure, freedom, feedback, depth, breadth, and further exercises and examples that the child needs to master the concept being studied. A typical lesson in a mathematics classroom takes on a [hypothetical learning trajectory](#) (Simon, 1995), and by listening to the students, we can slightly, or radically, alter that trajectory. This article will focus on a [taxonomy of listening](#) developed by Davis (1997), and includes three different, yet co-implicated ways to listen in a mathematics classroom; evaluative listening, interpretive listening, and hermeneutical listening. Each of these will be accompanied by some ideas on how the trajectory of the lesson evolves based on what is heard in each listening style.

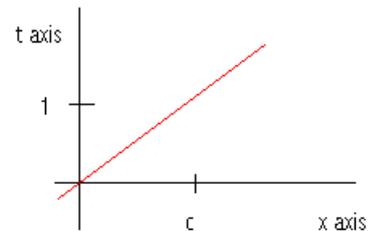
### Evaluative Listening

The purpose of listening at this level is to assess what is being studied. In other words, to check for right or wrong answers, or to find something that the teacher is trying to correct. In this stage, the teacher is listening to the *mathematics*, and not to the *student*. An example of this level of listening would be when a teacher asks a question and already has the answer in mind, for example what is 7 divided by 1. From a practical standpoint, the teacher is listening only for the answer 7, and not to the underlying sense making of the student. The answer is always 7, and needs no further explanation.

This type of listening has its place in the math classroom. It is an effective way to check to see if all students are on the same page. However, this type of listening also

tends to be limited in terms of deeper mathematical understanding, and limiting in progressing that understanding to a more robust level. In terms of Simon's (1905) hypothetical learning trajectory, it may represent a missed opportunity to delve into issues and questions that would benefit the entire collective. By focusing on the simple mathematical rule that the dividend divided by the divisor equals the dividend when dividing by 1, opportunities to delve into the larger issues and processes behind division are missed. This is done for the immediate benefit of staying on course of the predetermined lesson plan. In this regard, the student contribution has little or no effect on the trajectory of the lesson. When learning is streamlined and standardized for redundancy between all members of the class, the context of the group and where they are in the world and society play little role in the lesson trajectory. The teachers listening is now about making sure that the students are listening to the teacher, and not about challenging and perturbing their sense of mathematics. The lesson would appear to take the shape of a straight line, moving from what is sought after, to what is correct.

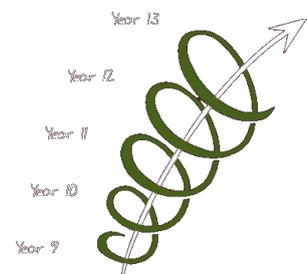
Mathematics itself becomes a discipline that is about converging on a set of true or correct understandings, and ultimate authority and access to the world of mathematical understanding lies with the teacher.



## Interpretive Listening

In this level of listening, the teacher is looking for the interpretation that the student is bringing to their answer. Through a constructivist understanding, the teachers main impetus here is to listen for an embodied history and make a preemptive interpretation regarding their meaning and implicit understandings behind their response. Questioning and dialogue with students now moves from the right/wrong of evaluative listening, into how/why do you think that. The main role of the teacher is seeking information. This is done primarily through more elaborate explanations and demonstrations. In the example of division by 1, it may be done by asking the students to show a visual diagram of dividing by 1, or to manipulate blocks or cubes. In this example, the student is now attempting to show that by dividing something by 1, you are not actually making any cuts or changes to the original dividend. A much different sense of mathematics than in the evaluative listening level.

As for the hypothetical learning trajectory of the lesson, interpretive listening may lead to answers or explanations that the teacher could not fully anticipate, and open up the space of what is possible (Davis, 2008). This opening up allows for a new direction and re-presentations of concepts, revisions of ideas, and side forays into related concepts. It is unlike evaluative listening in the sense that the trajectory may not be as linear, and may move around the main subject, while always being anchored to it, more like a spiral than a straight line. Part of the necessity for these detours is the awareness on the teachers part of the power of language and establishing a common, taken as shared vocabulary with the collective that will allow the group to move forward. In the case of interpretive listening as illustrated by Davis (1997), the teacher is mindfully aware of this necessity, and is adjusting he trajectory to specifically make the language accessible to all. The meaning of the vocabulary is not imparted from the teacher, but rather is settled through discussion and collaborative sense making, or phrased differently, the communication is negotiable.



Yet, in terms of the questions *what is mathematics*, the students are still attempting to converge on a true or correct set of understandings about the concepts being studied. Mathematics is still linear, and authority and access to the mathematics still lies with the teacher.

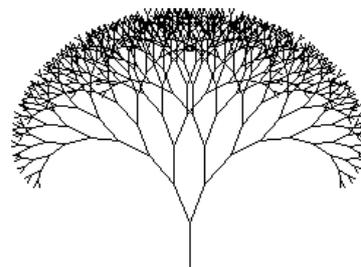
## Hermeneutic Listening

Hermeneutic listening is all about seeking variation and participating in making meaning. In this level, the class members and the teacher are jointly exploring a math concept and co-developing new ways of seeing the idea, rather than mastering already fragmented pieces of the concepts. The first two levels of listening treated math as an attainable and solid foundation of knowledge, transcendent and independent of the learners. Metaphorically, the students were simply trying to figure out how to reach out and grab the ring. In the level of hermeneutic listening, the mathematics itself is more fluid, the unanticipated plays a major role, and intuition and feeling are noticeably present. The collective is deconstructing the concept with the hope that they will put it back together in a way that makes sense to them. This has been described as *Concept Study* by Davis and Renert (2010). As for the teacher, since this is unexplored water and a fluid environment, the teacher moves from being the holder of mathematical knowledge, to a co-participant who is also deconstructing their own understanding alongside the learners. To take our example of dividing by 1 further, the students could now analyze and decompose the word divide. If the dividend is not being cut or altered into new pieces, are we really dividing? This type of question would lead to new and emergent insights and would perturb the learners sense of division.

The hypothetical learning trajectory in a lesson like this would appear from the outside to be messy and unstructured. Yet, there is more happening here than just the realizations and entailments of a mathematical concept. The group dynamics are leading the lesson, and the collective is acting as a unified whole, making sense that would not have been possible by one individual agent.

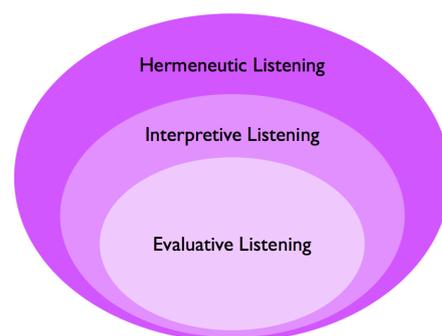
There is a joining of the collective on the sense making level, with the teacher as a participant. This type of lesson would be more fractal in nature, as the end result is unknown until it is arrived at, and each different group would come to varying end points within the fractal tree.

Mathematics then is no longer a knowable object, and no longer are we trying to just develop skills with certain concepts. Rather, we are transforming and reforming our individual and collective understanding of what mathematics is.



## Nested Levels and Mindful Awareness

This is not to suggest that all mathematics lessons should follow the fractal path of hermeneutic listening. All three levels are present, and are co-implicated (nested) in each other. One level is not abandoned for another, but rather is up to the teacher to be mindfully aware of when to move to different levels (or to be aware when the collective has moved to a different level), and to choose their learning trajectory wisely. A classroom teacher that only listened hermeneutically would miss a large skill base for their students, just as a teacher who was only listening with an evaluative ear would present mathematics as a closed system of thought.



## 2) Probes and Prompts

*Examples of different levels of questions*

### Response seeking questions (Evaluative Listening)

- If you take 3 apples and add 2 more, how many apples do you have?
- What is the square root of 16?
- If we know this side of the square is 5cm, what is the perimeter of the square?



### Information seeking questions (Interpretive Listening)

- Could you explain that another way?
- How did you get this number?
- Why did you decide to do it this way?

### Transformation seeking questions (Hermeneutic Listening)

How would you define a fraction?  
Does this remind you of anything else?  
How do you *see* that?

## 3) Taking Action

During your lessons, keep notes about when you are switching between levels; who initiated the switch, the learner or the teacher? What was said right before the switch took place? Reflect on those switches and whether or not they helped the class move forward.

Ask students to reflect on what they thought the lesson was about, and answer the same question yourself. Compare your answer to theirs.

Before the lesson starts, draw a line with how you think the lesson will unfold and what the trajectory will look like. While the lesson is unfolding, draw lines in a notebook and mark the points where the lines moved or changed. How close to the original lesson trajectory did you end up? Why? Is the difference or similarity a positive or a negative from the students perspective?

Record your lesson and watch it back, with or without the students. What questions did you gloss over? At what points did the lesson alter dramatically from your original plan?

## 4) References

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