Structured Variance
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Common Multiples

Basic Assumptions:
Students have experienced multiples (finding, examining patterns, predicting, interpolating and extrapolating) in hundreds charts. Students have experienced multiplication and multiplication families of these given number. They have a linguistic familiarity with words like pattern, multiple, common, etc.

- I am keeping the number 3 invariant (Mason and Watson, 2006) because there are too many examples of 2 (from Anne's lovely poem, 2012), and the lowest prime number **(3)** will give us options within our dimensions of possible variation (Mason and Watson, 2006).
- Additionally, within the dimensions of possible variation we constrained the task to the following values:
    - **(2)** when paired with 3, affords a large number of common multiples and suggests a relationship of "6-ness" to support conjectures.
    - **(5)** less common multiples, which draws a more specific attention to the relationship between 3 and 5.
    - **(4)** when paired with 3, affords still a reasonably large number of common multiples to support conjectures, and may suggest a relationship to "12ness" and a connection to 2, 3 and 5.
    - **(6)** provides the opportunity to look at the "special" case of common multiples when the second value is already a multiple of the first.
- Each stage will scaffold the next by further clarifying the relationship between what is happening, and developing a rule that will explain the unfolding pattern.
- The final stage will challenge that rule and open a new space of possibilities in which to explore (Davis, 2008)

1) On the first hundreds chart locate and mark the multiples of 2. What do you notice? What numbers are in common? Hypothesize a rule for what is happening.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples 3/2

- The intentionality behind this rational is to see how the common multiples are growing. This will more than likely be done by assigning a value to the difference of the green squares in the form of an informal rule. For example, every 6 squares there is a common multiple.
- Students may or may not notice that 6=2x3, and at this point it is not essential that they do, though that conjecture would be welcome. The grounding point for this stage is a sense of 6-ness.

Common Multiples

2) On the second hundreds chart locate and mark the multiples of 5. What do you notice? What numbers are in common? Adjust your hypothesis if needed. How is this like the previous chart?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples 3/5

- This step will invite them to adjust their hypothesis (common square every 15 boxes)
- By asking them to compare this chart to the last, we are encouraging them to think of a more formal rule to describe what is happening, with the hope that they will notice the pattern of (15 = 3 x 5) and (6 = 2 x 3)
- It may or may not emerge as a formal rule, but they will be grounded in a sense of 6-ness *and* 15-ness, elaborating and scaffolding on the previous chart

3) On the third hundreds chart locate and mark the multiples of 4. What do you notice? What numbers are in common? How does this chart compare to the previous charts? Hypothesize a rule that sums up all charts.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples 3/4

- They may notice that this chart is common every 12 squares. By asking them to step back see the pattern from as a whole, I am hoping that they notice the following pattern (see table), and orient attention to the relationship between the three charts. The sense of 6-ness, 15-ness, and 12-ness will afford them an opportunity to construction a formal rule. The rule stipulates that by ***multiplying the multiples together and then skip counting forward by the product they can reach any multiple of those two numbers***

Multiples	Common Multiples every (n) squares
2, 3	6
3, 5	15
3, 4	12

4) On the fourth hundreds chart locate and mark the multiples of 6. What do you notice? What numbers are in common? Does this fit your hypothesis? If not, explain what is different.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples 3/6

- This step is the first challenge to their formalized rule, and will re-orient them to think deeper about the relation between multiples. Applying the previous rule does not work in this case (common square every 12 squares, but 3 x 6 ≠ 12).
- This will open further inquiries that may enable them to see that if the two multiples are multiples of each other, then the rule is different
- This idea would be tested in further investigations

Reflection

Several aspects of common multiples became explicit for me during this process. The idea that common multiples can be expressed as their product with skip counting was a new way of seeing the concept. I believe this is something that I implicitly understood, but the phrasing of it in this regard will be beneficial to my classroom practice. These charts will serve as a powerful didactic object for my students (Thompson, 2002). The process of coming to this realization afforded me the power of testing conjectures in a scaffolded process, while also challenging those conjectures and opening a new space of possible investigation. This hypothetical learning trajectory (Simon and Tzur, quoted in Mason and Watson, 2006) started off as a linear process, but ended not with a final point along the line, but rather an opening of new directions. This was something I struggled to understand this week, but this activity has given me an embodied experience of it in action.

The collective aspect of this task was an intriguing experience. The collaboration and discussion between Mike, Jenn, and myself led to an interesting space of variation nested within the task. We all focused on different affordances, limitations and dimensions of possible variance, but all three different ways of knowing have allowed us to see a larger picture. For me, the collective scaffolding involved in creating this task was an incredible exercise that pushed me to think deeply about every number, and what order in which they appeared. For instance, I was debating with myself and my partners whether or not I should introduce the 3/5 chart before the 3/4 chart or vice versa. Both led to different affordances and different ways of discerning. I found myself creating conjectures, testing them, and either justifying them or revising them. Embedded within the creation of this lesson are the habits of mind of mathematical thinking (Cuoco, 1997).

As for the collective understanding of our group, it is fascinating to see how our three variations of the same task are part of one integrated whole.

References

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